# An Experimental Study on Measurement of Poisson's Ratio with Digital Correlation Method

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SYNOPSIS

A newly developed optical testing method, the digital correlation method (DCM), is discussed in this paper, and its use in testing of Poisson's ratio of nonmetal, low modulus viscoelastic materials. DCM is a noncontact testing method and is very easy to adapt to the environment. It has very bright prospects for wider applications. © 1996 John Wiley & Sons, Inc.

## INTRODUCTION

Poisson's ratio, as one of basic mechanical property parameters of materials, is very important in many finite element method (FEM) calculations, and its precision directly affects the reliability of computation results. There have been some mature methods to test metal's Poisson's ratio, such as electric gauge, speckle interferometry, and holography. But for some viscoelastic nonmetal materials such as solid rocket propellant, it is very difficult to get good results with these methods because of their different mechanical characteristics from metals. Solid propellant has a loose structure and low elastic modulus (only about 200 MPa), and its surface is very difficult to be machine-shaped to a smooth surface. So if the electric gauge method is adopted, the foil gauges will add additional stiffness to a specimen, which cannot be neglected. And there will be some gaps between foil gauge and specimen surface. These will greatly decrease the precision of testing. If speckle interferometry or holography is adopted, then the deformation of propellant can reach to so high a level that interference fringe patterns will become too dense to be processed, even when the load is quite low. In addition, experiments have to be done in a darkroom, so the time effects of materials deforming also add some difficulties to double exposure. The volume-change method cannot be employed in most labs because of the complicated equipment required. There are also problems in the newly developed method to measure propellant's Poisson's ratio, the fringe scanning method.[1] For instance, longitudinal and transverse strains cannot be measured in the same setup and high level machine shaping of a specimen surface is required. The digital speckle correlation method (DCM or DSCM)[2] has very good noncontact testing characteristics, and quite simple experimental conditions are required: no darkroom is required and no special machine shaping is required to a specimen surface. And under many circumstances it can even be employed in the field. DCM has been used in the measurement of plane displacement and strain. Also, DCM has some shortcomings; extensive calculation work and much calculation time are needed. Often it is executed on advanced or fast computers, a VAX, for example. These limit wider application of DCM to some degree. So in this paper, DCM is improved according to these points and is used in testing of solid propellant Poisson's ratio.

## BASIC PRINCIPLES

The essential point of DCM is to get the whole-field distribution of displacement and strain by studying the distribution of the digital gray field of the object before and after its deformation. The testing process is as follows: First, photograph two images of the object before and after its deformation. Then assume a displacement field and a strain field acting on a point of the object, add the displacement deviation to the point reacted to the displacement and strain

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field, find the corresponding point on the deformed image, and calculate the correlation function value between the two points. Then among the many assumed fields, the real displacement and strain fields should make the correlation function reach the maximum value.

As shown in Figure 1, object $R$ becomes $R^*$ after deformation. $P$ is an arbitrary point on $R$, and $Q$ is another point near to point $p$ on object $R$. After deformation, $P$, $Q$ arrive at $P^*$, $Q^*$ on $R^*$. The vector from $P$ to $P^*$ is called the displacement vector of this point, written as $\mathbf{u}$ or $\mathbf{u}(P)$

$$u_i = x_i - X_i = 1, 2, 3 \tag{1}$$

in which $X_i$, $x_i$ represent point $P$'s position coordinate before and after deformation, respectively.

After deformation point $Q^*$ should still be quite near to $P^*$ according to the continuously differentiable characteristic of a nonindividual body. The vector from $Q$ to $Q^*$, i.e., displacement of point $Q$ is

$$\mathbf{u}(Q) = \mathbf{u}(P) + d\mathbf{u} = \mathbf{u} + d\mathbf{r}\cdot\nabla\mathbf{u} \tag{2}$$

or we can write it as

$$u(Q) = u + \frac{\partial u}{\partial x}dx + \frac{\partial u}{\partial y}dy + \frac{\partial u}{\partial z}dz$$

$$v(Q) = v + \frac{\partial v}{\partial x}dx + \frac{\partial v}{\partial y}dy + \frac{\partial v}{\partial z}dz$$

$$w(Q) = w + \frac{\partial w}{\partial x}dx + \frac{\partial w}{\partial y}dy + \frac{\partial w}{\partial z}dz \tag{3}$$

As to plane problems, $w = 0$, and we replace $dx$, $dy$ with $\Delta x$, $\Delta y$, then eq. (3) can be written as

$$u(Q) \approx u + \frac{\partial u}{\partial x}\Delta x + \frac{\partial u}{\partial y}\Delta y$$

$$v(Q) \approx v + \frac{\partial v}{\partial x}\Delta x + \frac{\partial v}{\partial y}\Delta y \tag{4}$$

$u$, $v$, $w$ in eqs. (3) and (4) are the displacements in the direction $x$, $y$, $z$, respectively. Strains can be expressed as follows according to the small deformation principle in elasticity theory.

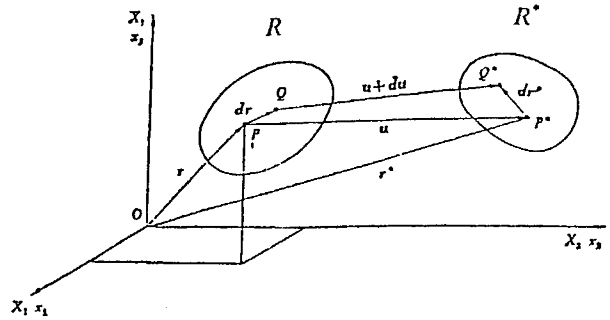


**Figure 1** Three-dimensional coordinate system.

$$\varepsilon_x = \partial u/\partial x,\ \varepsilon_y = \partial v/\partial y$$

$$\varepsilon_z = \partial w/\partial z$$

$$\varepsilon_{xy} = \varepsilon_{yx} = \tfrac{1}{2}(\partial u/\partial y + \partial v/\partial x)$$

$$\varepsilon_{xs} = \varepsilon_{zx} = \tfrac{1}{2}(\partial u/\partial z + \partial w/\partial x)$$

$$\varepsilon_{yz} = \varepsilon_{zy} = \tfrac{1}{2}(\partial v/\partial z + \partial w/\partial y) \tag{5}$$

Because the gray field distribution on the object surface is random, we should take a subset with a certain size and take the average characteristics of all points in the subset as the characteristics of the center point. The subset should be big enough to contain enough random points, and small enough to make the strain in the subset uniform. So the displacement of every point in the subset can be described with the displacement of the center point and the strain (displacement derivative) in the subset.

If an arbitrary subset on the image before deformation is given, and six parameters $u$, $v$, $\partial u/\partial x$, $\partial u/\partial y$, $\partial v/\partial y$, $\partial v/\partial x$ are assumed, in which $u$, $v$ are the displacements of the center point of the subset, then the position of an arbitrary point $Q$ in the subset before and after deformation can be expressed as

$$x^* = x + u(Q) = x + u + \frac{\partial u}{\partial x}\Delta x + \frac{\partial u}{\partial y}\Delta y$$

$$y^* = y + v(Q) = y + v + \frac{\partial v}{\partial x}\Delta x + \frac{\partial v}{\partial y}\Delta y \tag{6}$$

in which $\Delta x$, $\Delta y$ are the components in the $x$, $y$ directions of the vector between point $P$ and $Q$. According to eq. (6), on the deformed image we can find the subset corresponding to a given undeformed subset (see Fig. 2).

The digital image taken from a camera is the result of discretion of the actual gray field, and gray level values exist only at integral coordinate points

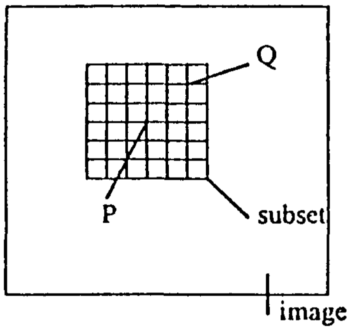


**Figure 2** Scheme for image subset: $P$ is the center point of the subset and $Q$ is an arbitrary point in the subset.

(dimension: pixel). So the digital gray field of the deformed image has to be interpolated. There are many forms of interpolation calculation, and the simplest one, bilinear interpolation, is shown in Figure 3. Assume a point $Q(i+y, j+x)$ that is just among four integral pixel points $P(i, j)$, $P(i+1, j)$, $P(i, j+1)$, $P(i+1, j+1)$, in which $x$, $y$ are the fractional parts of the position coordinates, and $0 \le x < 1$, $0 \le y < 1$, then the gray level value of point $Q$ can be derived by interpolating the gray level values of the surrounding four points.

$$\begin{aligned} Q(i+y, j+x) &= G(i,j)(1-x)(1-y) \\ &\quad + G(i,j+1)(1-y)x + G(i+1,j)(1-x)y \\ &\quad + G(i+1,j+1)xy \end{aligned} \tag{7}$$

in which $G(i, j)$, $G(i+1, j)$, $G(i, j+1)$, and $G(i+1, j+1)$ represent the gray level values of the four points. The two gray level fields corresponding to the two image subsets before and after deformation can be arranged as two one-dimensional arrays $P_1$, $P_2$ in a certain same form and sequence. So a two-dimensional correlation calculation problem can be simplified into a one-dimensional linear correlation problem. Assume $X$, $Y$ to be two random variables, and in probability statistics, the correlation ratio between $X$, $Y$ can be defined as

$$\begin{aligned} \rho(X, Y) &= \frac{\operatorname{cov}(X, Y)}{\sqrt{V(X)V(Y)}} \\ &= \frac{E\{[X - E(X)][Y - E(Y)]\}}{\sqrt{V(X)V(Y)}} \end{aligned} \tag{8}$$

in which $E(X)$, $E(Y)$, $V(X) > 0$, $V(Y) > 0$ are the averages and square deviations of $X$, $Y$.

Two one-dimensional image arrays can be regarded as two one-dimensional discrete random variables. The discrete points number is $m$, and $m = (n+1)(n+1)$, $n+1$ is the length of side of the image subset.

$$E(P_1) = \frac{1}{m}\sum_{i=0}^{m-1} P_1(i)$$

$$E(P_2) = \frac{1}{m}\sum_{i=0}^{m-1} P_2(i)$$

$$\begin{aligned} V(P_1) &= E(P_1^2) - [E(P_1)]^2 \\ &= \frac{1}{m}\sum_{i=0}^{m-1} P_1^2(i) - \left[\frac{1}{m}\sum_{i=0}^{m-1} P_1(i)\right]^2 \\ V(P_2) &= E(P_2^2) - [E(P_2)]^2 \\ &= \frac{1}{m}\sum_{i=1}^{m-1} P_2^2(i) - \left[\frac{1}{m}\sum_{i=0}^{m-1} P_2(i)\right]^2 \end{aligned} \tag{9}$$

The correlation ratio between $P_1$ and $P_2$ is

$$C = \frac{(1/m)\sum_{i=0}^{m-1} [P_1(i) - E(P_1)][P_2(i) - E(P_2)]}{\sqrt{V(P_1)V(P_2)}} \tag{10}$$

$P_2$ actually is the function of displacement and its derivative, so we call $C$ the correlation function between $P_1$ and $P_2$.

$$C = C(u, v, \partial u/\partial x, \partial u/\partial y, \partial v/\partial y, \partial v/\partial x) \tag{11}$$

The simulation degree between object surface gray fields before and after deformation can be described with correlation function $C$. As to a certain point $(x, y)$ before deformation, when we change the six parameters $u$, $v$, $\partial u/\partial x$, $\partial u/\partial y$, $\partial v/\partial y$, $\partial v/\partial x$, we can get different correlation function values. When the correlation values reach a maximum value, then

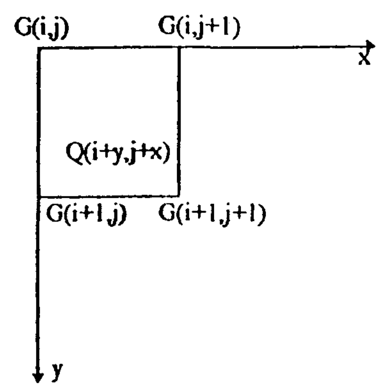


**Figure 3** Bilinear interpolation of image gray field.

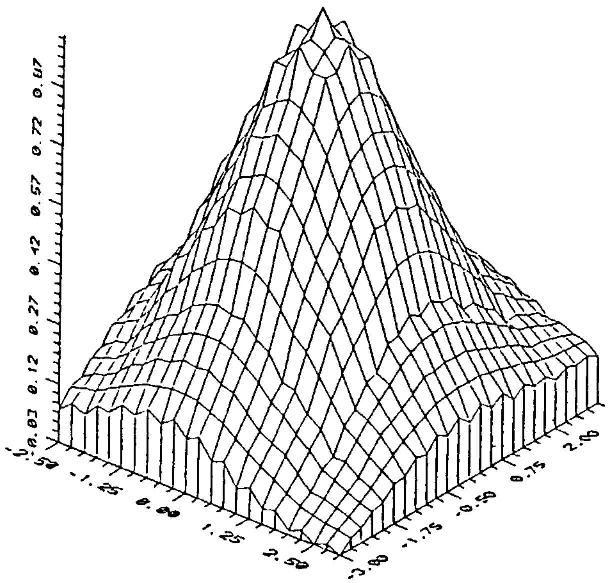

**Figure 4** Unipeaked characteristic of correlation function.

the displacement and derivative can be regarded as the actual displacement and derivative at this point. Measuring displacement and its derivative with DCM allows one to find the group of displacements and derivatives that can make the correlation function reach a maximum value. It is shown by our experiments that the correlation function given by eq. (10) is most sensitive to the changes of $u$, $v$, and not very sensitive to the changes of derivatives. In the two-dimensional function space $u - v$, $C(u, v)$

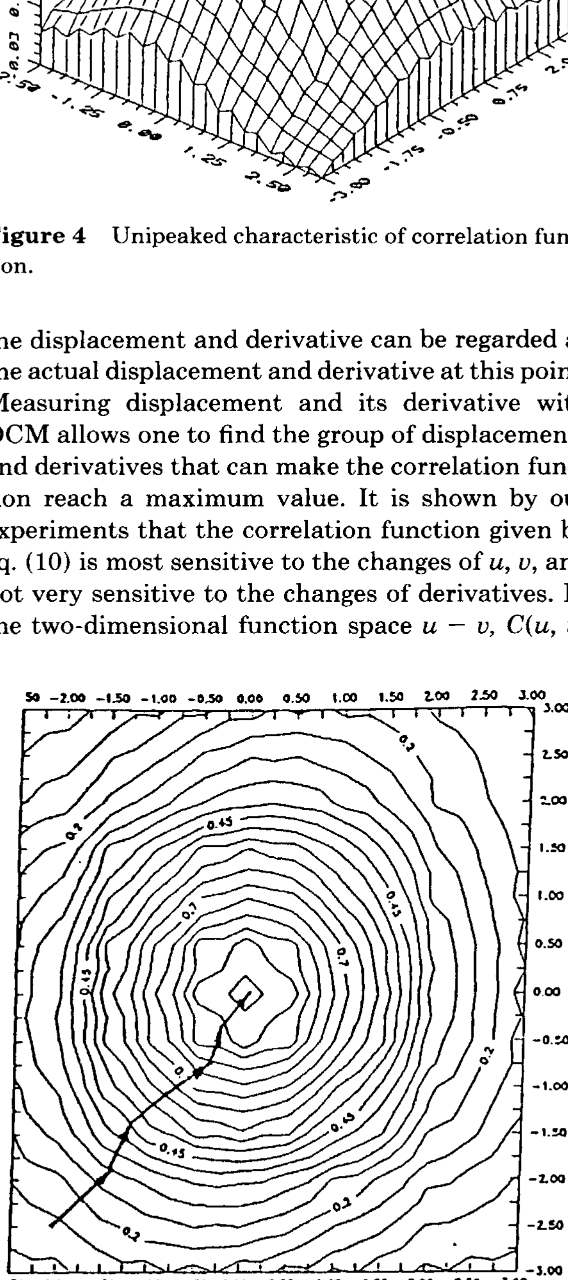


**Figure 5** Searching path of climbing-hill method.

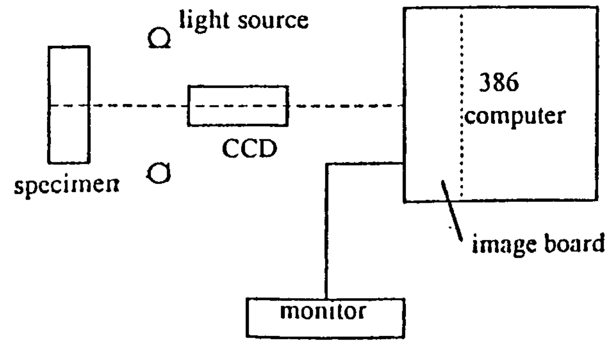


**Figure 6** Experimental setup for DCM.

demonstrates very good unipeaked characteristic (Fig. 4). The correlation function will quickly decrease when $u$, $v$ move in a direction far away from actual displacements. But in the six-dimensional function space, there may be some peak values for $C(u, v, \partial u/\partial x, \partial u/\partial y, \partial v/\partial y, \partial v/\partial x)$, which are very near to each other, then we can get only relative maximum values. In a given searching region, we can always find the maximum value if we can gradually reduce the searching step and study the correlation function values at every displacement and its derivative combination. That is the method often used before in many papers that is called *coarse and fine*. This method is very time-consuming and unacceptably long if executed on a personal computer.

DCM is improved from the view of optimal design in this paper. First let $u = v = \partial u/\partial x = \partial u/\partial y = \partial v/\partial y = \partial v/\partial x = 0$, and search in $u - v$ space to find the best point; and then keep the value of $(u, v)$, $(\partial u/\partial y, \partial v/\partial x)$ and search the best point in $(\partial u/\partial x, \partial v/\partial y)$ space, and then keep the value of $(u, v)$, $(\partial u/\partial y, \partial v/\partial x)$ and search the $(\partial u/\partial x, \partial v/\partial y)$ space. Iterate like this until a given precision is reached. During every step of searching and iteration, a direct searching method, the climbing-hill method, is adopted (Fig. 5). Eight directions and eight direction points around every given point are studied and the direction along which the correlation function gradient is the greatest is selected and then the following searching is done along this direction. The calculation work of this method is much less than *coarse and fine*. Our experiments show that calculation speed can be increased 8 times, and it can be run on a personal computer.

The derivative of displacement obtained this way is very small relative to displacement, and its precision is quite low. So we do not use these derivative values directly, but compute derivatives by fitting the displacement values. The reason why we con-

**Table I Testing of a Still Object with DCM**[a]

| Testing Points | $u$ (pixel) | $v$ (pixel) | $\partial u/\partial x$ (%) | $\partial v/\partial y$ (%) | $\partial v/\partial x$ (%) | $\partial u/\partial y$ (%) | Correlation Function |
|---|---|---|---|---|---|---|---|
| (255, 255) | 0.003 | 0.001 | 0.01 | −0.01 | 0.01 | 0.01 | 0.995 |
| (280, 260) | 0.002 | 0.001 | 0.01 | 0.02 | 0.01 | 0.02 | 0.991 |
| (230, 230) | 0.003 | 0.001 | −0.02 | 0.01 | 0.02 | 0.01 | 0.989 |
| (320, 300) | 0.001 | 0.002 | 0.0005 | 0.01 | −0.01 | 0.01 | 0.997 |

[a] 30 × 30 subset is used. Image recognition is 1 pixel = 0.05431 mm.

sider derivative values during searching and iteration is to increase correlation function values.

## EXPERIMENTS

The setup for DCM to measure plane displacement and strain is shown in Figure 6. The image card has 1 MB extended memory, and four 512 × 512 pixels images can be stored and visited. The gray level is 0–255 and the image snapping frequency is 25 images per second. A 33-MHz 386 personal computer (with math coprocessor) is used.

The specimen surface is smeared with vitreous speckles to be a good diffuse reflection surface. Common lamps can be chosen as the light source, but our experiments show that the light source in a slide projector is better in increasing correlation function values.

To estimate the error of the testing system, we photograph several images of the object when it is still. Theoretically, displacement and its derivative should be zero, but because of some disturbance conditions such as instability of the light source, electric noises of CCD camera etc., the gray level fields of object surface may be different in the images and, consequently, calculated displacement and derivative may not be zero. This can be regarded as a deviation of the testing system and, according to this, testing results can be corrected if necessary (see Table I).

Compression and tensile experiments are completed to test a solid propellant's Poisson's ratio. To get the Poisson's ratio at a given point, displacements $u_i$, $v_i$ ($i = 1, 2, \ldots, m$) at every point in the row and column passing the given point are measured with DCM ($m$ is the point number in a selected row or column).

In both transversal and longitudinal directions, displacements are fit in the form of polynomials, and the derivative at the given point is derived from the polynomial. So strains in two directions $\varepsilon_x$, $\varepsilon_y$ are obtained (Fig. 7). According to the definition of Poisson's ratio, we can get

$$\mu = -\frac{\varepsilon_x}{\varepsilon_y} \text{ (loading direction is along } y \text{ axis)} \qquad (12)$$

**Table II Tensile Test Result of a Certain Propellant, Testing Region**[a]

| Time (s) | $t = 0$ | $t = 3$ | $t = 6$ |
|---|---|---|---|
| 1 | 0.4594 | 0.4886 | 0.4779 |
| 2 | 0.4612 | 0.4860 | 0.4728 |
| 3 | 0.4663 | 0.4775 | 0.4811 |
| 4 | 0.4725 | 0.4915 | 0.4692 |
| 5 | 0.4688 | 0.4803 | 0.4714 |
| 6 | 0.4636 | 0.4869 | 0.4672 |
| Average | 0.4638 | 0.4862 | 0.4745 |
| Standard deviation | 0.0055 | 0.0051 | 0.0053 |

[a] 7.2 × 7.2 mm, 30 × 30 pixel subset is used. Image recognition 1 pixel = 0.0534188 mm.

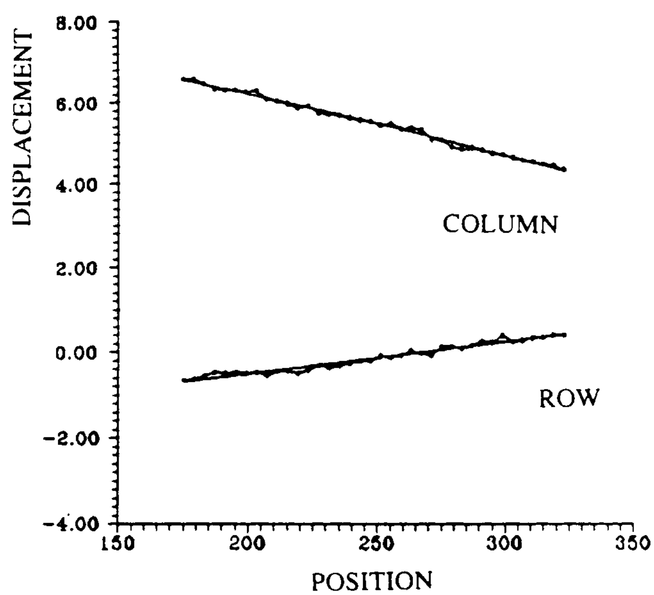


**Figure 7** Compression test of a certain propellant: $\varepsilon_x = 0.0075879$, $\varepsilon_y = -0.00153955$, $\mu = 0.493$.

In a small region around a point ($3 \times 3\ mm^2$) several points are tested. The average of the Poisson's ratio values at the chosen points can be taken as the Poisson's ratio of the given point, i.e.,

$$\bar{\mu} = \frac{1}{n} \sum_{i=1}^{n-1} \mu_i \tag{13}$$

in which $n$ is the number of testing points. Testing precision can be estimated by standard deviation

$$\sigma = \sqrt{\frac{1}{n-1} \sum_{i=1}^{n-1} (\mu_i - \bar{\mu})^2} \tag{14}$$

Experiments show that $\sigma$ is less than 0.005, so relative precision can reach to 1% (see Table II).

## CONCLUSION

The setup and experimental conditions required for DCM are very simple. For nonmetal, large deformation problems, DCM can get quite good results.

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